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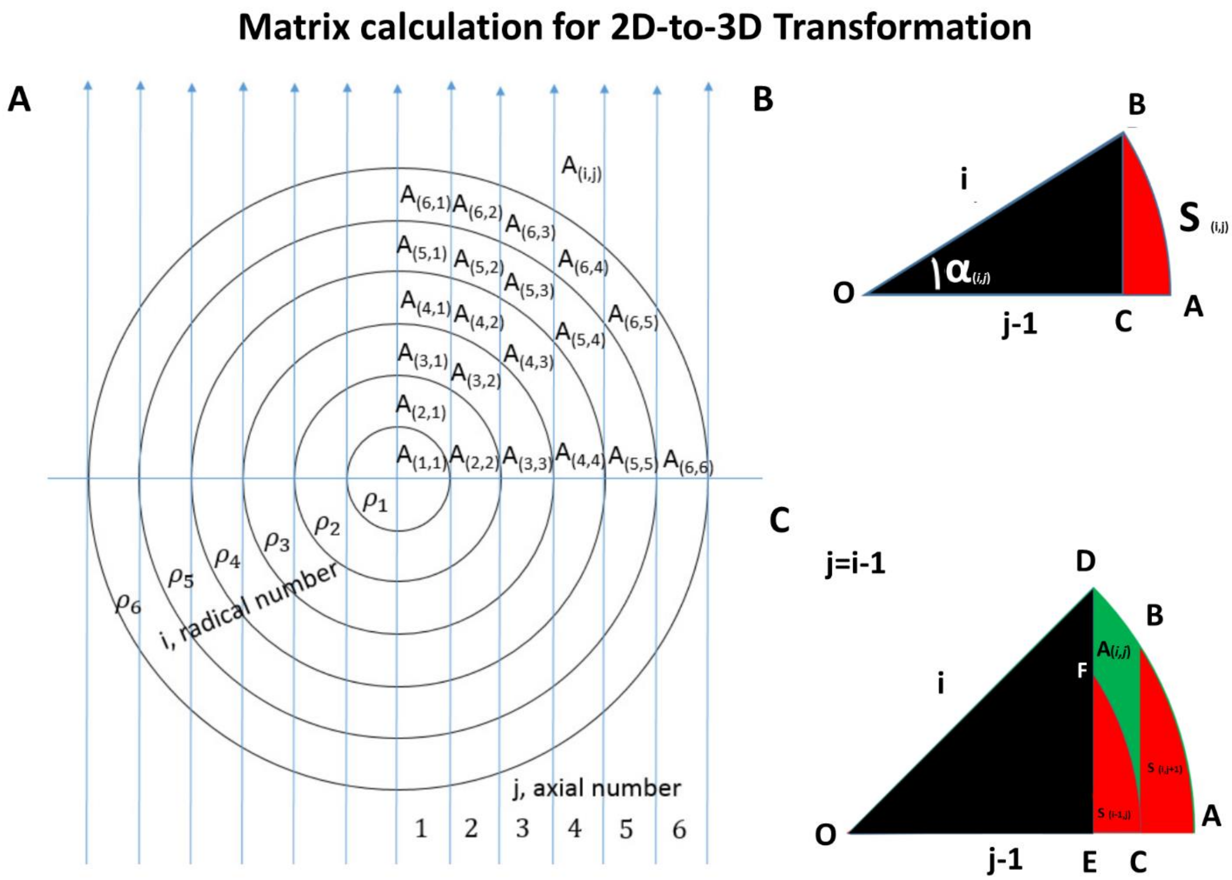
Supplementary information

High-speed super-resolution imaging of rotationally symmetric structures using SPEED microscopy and 2D-to-3D transformation

In the format provided by the authors and unedited

Supplementary material



Supplementary figure 1: ***Determining the area of each subsection in the 2D-to-3D transformation area matrix.*** A) Labeled area matrix which reflects the contribution of each ring to the 2D distribution. We define i as the radial number and j as the axial number. The density of single molecule locations along radius i is assumed to be uniform given the radial symmetry and the density is labeled as ρ_i here. The cross-section of radial number i and axial number j is defined as A(i,j). In the following equations, Δr is the bin size, ρ_i is the density of molecules in the ith ring, h is the half of illumination depth and ρ is the constant background density outside the region of interest. B) To determine the area of each sub-region A(i,j), it is necessary to calculate the area of the fan-shape area at j=i, in which:

$$\cos \boldsymbol{\alpha}_{(i,j)} = \frac{j-1}{i} \qquad \sin \boldsymbol{\alpha}_{(i,j)} \frac{\sqrt{i^2-(j-1)^2}}{i} \qquad \boldsymbol{\alpha}_{(i,j)} = \cos^{-1}\frac{j-1}{i}$$

The red-shaded area is defined as S(i,j):

$$S_{(i,j)} = S_{ABC} = S_{AOB} - S_{BOC}$$

$$S_{(i,j)} = \frac{1}{2}\alpha_{(i,j)} i^2 \Delta r^2 - \frac{1}{2}\sin\alpha_{(i,j)} \cos\alpha_{(i,j)}\, i^2 \Delta r^2$$

$$S_{(i,j)} = \frac{1}{2}\left[\cos^{-1}\frac{j-1}{i}\right] i^2 \Delta r^2 - \frac{1}{2}\left[\frac{\sqrt{i^2-(j-1)^2}}{i}\right]\left(\frac{j-1}{i}\right) i^2 \Delta r^2 \quad \textbf{(Eq.1)}$$

C) When j ≠ i and i > j, we need to calculate the area of the green-shaded region as follows:

$$S_{CBDF} = S_{ADE} - S_{CFE} - S_{ABC} \qquad S_{GMRP} = S_{AQP} - S_{CQR} - S_{AHG} + S_{CMH}$$

$$A_{(i,j)} = S_{(i,j)} - S_{(i-1,j)} - S_{(i,j+1)} + S_{(i-1,j+1)} \quad \textbf{(Eq. 2)}$$

When i < j, $A_{(i,j)} = 0$ **(Eq. 3)**

Following the above equations, all A(i,j) can be precisely calculated. Finally, N_j, the number of events detected in the jth column can be calculated with the following equation:

$$N_j = 2\{ \sum_{i=j}^{n} \rho_i A_{(i,j)} + [h\Delta r - \sum_{i=j}^{n} A_{(i,j)}]\rho \quad \textbf{(Eq. 4)}$$

As soon as N_j and A(i,j) are known, ρ_i will be calculated.